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EXPERIMENTAL-THEORETICAL ANALYSIS OF NONSTATIONARY INTERACTION OF DEFORMABLE IMPACTORS WITH SOIL

V. G. Bazhenov, V. L. Kotov, S. V. Krylov,
V. V. Balandin, A. M. Bragov, and E. V. Tsvetkova

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An experimental method for determining the force of resistance to penetration of a deformable impactor into soft soil was developed in the inverted formulation: the impactor and the target exchange roles and the necessary parameters of contact interaction are recorded in an immovable measuring rod (impactor). To verify the basic principles of this experimental technique, wave processes were analyzed numerically using a modified Godunov's scheme. The applicability of various models of soil deformation was studied, and the calculation results obtained were compared with experimental data.

Determining the contact forces and accelerations that arise during interaction of an impactor with a target is one of the most difficult tasks in studying shock-accompanying processes. At present, various analytical and numerical methods have been used to solve this problem (see, for example, [1–5] and references therein). Experimental methods are also important for solving such problems. They can be divided into two groups, depending on the manner of formulation of the problem: direct methods [1, 2, 4–10] and inverted methods [5, 7, 10–13]. In direct experiments, the main experimental parameters are recorded using high-velocity flash photography and flash radiography [1, 2, 6], special sensors located on the impactor [5, 8–10], and sensors located inside the target [4, 7]. Because in most of these methods [1, 2, 4, 6, 7], the penetration depth is the main parameter to be measured, it follows that to determine the accelerations and forces, one need to differentiate experimental dependences twice, which reduces the accuracy of the results obtained. Although special sensors located on impactors [5, 8–10] allow one to determine the accelerations and forces more precisely, they complicate the experiment because it is necessary to prevent sensor breakage during impactor acceleration and provide for a reliable connection between the sensor and the measuring equipment.

So-called inverted experiments are free of the above-mentioned disadvantages. In these experiments, the target and the impactor exchange roles. The medium to be studied enclosed in a container is accelerated to the required velocities by a gas gun and impacts on an immovable impactor fitted with sensors. Because the displacements of the impactor are small at the initial moment, measurements of the interaction parameters are considerably simplified. Laser interferometry [11], piezoaccelerometers [11, 12], and the measuring rod technique [5, 7, 13] have been used most widely to measure velocities and accelerations in inverted experiments.

The present paper describes a method for determining the forces that arise during interaction of an impactor with soil based on the measuring rod technique. This technique consists of the following steps. The container filled with soil is accelerated to the required velocities and impacts on the immovable head of a mold fixed on the measuring rod. The impact velocity and the properties of the rod material should be such that plastic strains do not arise in the rod. In this case, an elastic strain pulse $\varepsilon(t)$ is formed in the rod at the moment of impact. From records of this pulse in the measuring rod, the force F acting on the impactor during interaction with the medium can be determined by the well-known relation $F(t) = E\varepsilon(t)A$, where E is the elastic modulus of the rod and A is

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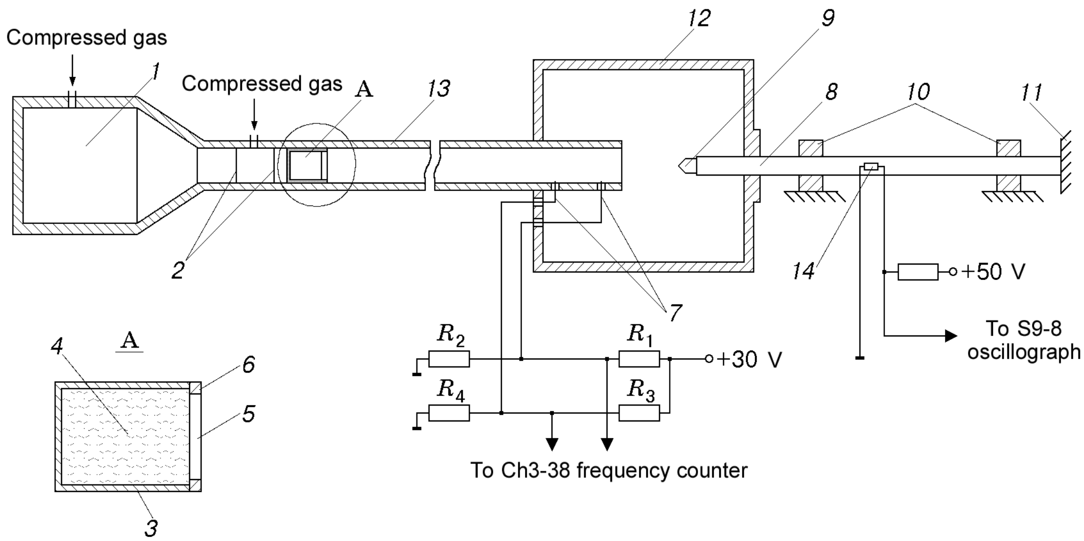

Fig. 1

the cross-sectional area. Thus, in this technique, the use of tensometry simplifies considerably force measurements, which reduce to recording the elastic strain pulse in the rod.

Figure 1 shows a diagram of the experimental setup employed in the method described above. In this modification of the inverted experiment, the container with soil was accelerated by a 0.57-diameter gas gun 1 with a two-diaphragm bolt 2, which operates with compressed air or helium at pressures of up to 15 MPa. The use of the gas gun allows one to obtain stable and easily controlled impact velocities in the range of 100–500 m/sec for masses of up to several hundred grams. The container 3 is a thin-walled cylinder of an aluminum alloy filled with soil 4. To prevent spilling of the soil during preparation of the experiment and acceleration of the container, the front part of the container was sealed with a 0.02-mm-thick Maylor film 5. The film was fixed and pressed to the soil surface by a vinyl plastic ring 6, which was tightly inserted into the container. The container velocity was determined by two electrocontact sensors 7 located in holes drilled in the gun barrel near the muzzle. The contacts were made of pieces of an insulated copper wire 0.5 mm in diameter and connected to a 50 V power source with the interposition of voltage dividers R_1/R_2 and R_3/R_4. The time interval between signals from the electrocontact sensors was recorded by a Ch3-38 frequency meter. The distance between the contacts was 200 mm; the accuracy of distance measurements was not less than 0.5 mm. The measuring rod 8 of 1.5 m length and 20.5 mm diameter was made of steel with a yield point of 1200 MPa. The left butt-end of the measuring rod had a threaded hole with a screw head 9 of the required shape. The rod was located at a distance from the muzzle so that impact occurred just after the departure of the container from the barrel. The support on which the rod was placed was fitted with positioners 10 to align the rod and the gun barrel and provide for axisymmetric interaction. The back butt-end of the rod rested at a special stop 11, which prevented displacement of the rod and absorbed impact energy. Impact occurred in a vacuum chamber 12 into which the gun barrel 13 and the measuring rod with the head were inserted.

Elastic pulses in the measuring rod were recorded by four KF4-P1 resistance strain gauges 14 connected in series and glued at a 90° interval on the lateral surface of the rod at 500 mm from the impacted end. The base of the resistance strain gauges was 3 mm. The strain gauges were powered by a potentiometric circuit [14] and a standard B5-30 power. The measuring channel was calibrated by a scale resistor of specified value [15] connected in series with the strain gauges. In addition, dynamic calibration of the entire recording system was performed on a setup with a Hopkinson split rod by exciting a strain wave of specified amplitude in the rod. The elastic strain wave amplitude was calculated by the well-known relation of one-dimensional theory $\varepsilon = V/(2C)$, where V is the measured impact velocity and C is the velocity of longitudinal waves in the rod. Information from the strain gauges was recorded by an S9-8 digital storage oscillograph. The experimental data were processed on a personal computer. The experimental method developed is used to measure the forces upon interaction of steel impactors with soft soil (sand, clay soil, loam, etc.).

Mathematical Model. The axisymmetric problem of the interaction of a cylindrical body with soft soil enclosed in a deformable casing is simulated. The dynamics of the interacting media and the laws of conservation of mass and momentum are defined by the equations

$$\rho_{,t} + (\rho u)_{,x} + (\rho v)_{,y} = -\rho v/y,$$

$$(\rho u)_{,t} + (\rho u^2 - \sigma_{xx})_{,x} + (\rho uv - \tau_{xy})_{,y} = -(\rho uv - \tau_{xy})/y, \tag{1}$$

$$(\rho v)_{,t} + (\rho uv - \tau_{xy})_{,x} + (\rho v^2 - \sigma_{yy})_{,y} = -(\rho v^2 - \sigma_{yy} - \sigma_{\theta\theta})/y.$$

Here t is time, ρ is the density, u and v are the velocity components in the x and y directions, respectively, and σ_{ij} are the stress-tensor components. The subscript after the comma denotes differentiation with respect to the corresponding variable. To describe the volume compressibility of soft soil, we used the equation

$$(\rho\rho_*)_{,t} + (\rho\rho_* u)_{,x} + (\rho\rho_* v)_{,y} = -\rho\rho_* v/y, \tag{2}$$

where ρ_* is the maximum density attained in the process of active loading of the soil. Equation (2) takes into account the load history of a material particle. The constitutive relation between the volume strain $\varepsilon = 1 - \rho/\rho_0$ and the soil pressure p, where

$$p = M\varepsilon^n, \tag{3}$$

in the range of $p = 0.2$–10 MPa is derived on the basis of experimental results [16]. The use of the shock adiabat obtained from results of experiments with plane waves [17–19] is justified for pressures above 250 MPa. The linear dependence between the shock-wave velocity D and the mass velocity behind the wave front U $D = A + BU$, using the Hugoniot relations, can be written as

$$p = \rho_0 A^2 \varepsilon/(1 - B\varepsilon)^2, \tag{4}$$

where ρ_0 is the initial soil density and A and B are constants. In the range of 10–250 MPa, we use an interpolating parametric cubic Bezier-type polynomial [20], which ensures continuity of the velocities of sound (the first derivative) at the junction points. The Bezier polynomial is written as

$$\boldsymbol{r}(w) = \{\rho(w), p(w)\} = (1-w)^3\boldsymbol{r}_1 + 3w(1-w)^2\boldsymbol{r}_2 + 3w^2(1-w)\boldsymbol{r}_3 + w^3\boldsymbol{r}_4. \tag{5}$$

With variation in w from 0 to 1, polynomial (5) in the (ρ, p) coordinates passes through the points (ρ_1, p_1) and (ρ_4, p_4), and the slope of the tangent at these points coincides with the slope of the tangents obtained from (3) and (4), respectively. Expressions for the densities ρ_2 and ρ_3 are given by

$$\rho_2 = 1 + \alpha\rho_1, \qquad \rho_3 = 1 - \beta\rho_4, \tag{6}$$

and the corresponding pressures are obtained by substitution of densities into the equations of the tangents. The equations of the tangents and the values of the polynomial at the reference points (ρ_1, p_1) and (ρ_4, p_4) are derived using dependences (3) and (4). Thus, the values of the function and its derivatives are determined uniquely. The concrete form of function (5) satisfying these conditions depends on the choice of values of α and β, and the substitution of these constants into (6) must ensure, at least, convexity and uniqueness of the interpolating polynomial. Unloading of the medium is described by the two-segment broken line [15]

$$p = \begin{cases} p^* + C_1^2(\rho - \rho^*), & p > p_{00}, \\ p^* + C_2^2(\rho - \rho^*), & p < p_{00}. \end{cases} \tag{7}$$

Here C_1 and C_2 are the velocities of sound that determine, respectively, the slopes of the first and second segments of the broken line (7) to the ρ axis, $p_{00} = p^*/\gamma_p$ characterizes the ratio of lengths of the segments of the broken line, (ρ_{00}, p_{00}) is the point of inflection on the rarefaction curve in the (ρ, p) coordinates. The velocities of sound C_1 and C_2 as functions of ρ^* are given by

$$C_1 = C_g + \frac{\rho_g - \rho^*}{\rho_g - \rho_0}\,(C_0 - C_g), \qquad C_2 = \frac{C_g}{\gamma_c} + \frac{\rho_g - \rho^*}{\rho_g - \rho_0}\Big(C_0 - \frac{C_g}{\gamma_c}\Big).$$

The parameter γ_c specifies the ratio of C_1 to C_2 for $\rho^* = \rho_g$, where ρ_g is the density at which the loading segment corresponding to inverse (hydrodynamic) loading begins. At the point (ρ_g, p_g), the slope of the first segment of the broken line (5) coincides with the slope of the tangent to the shock adiabat in (2). Thus, we have specified the linear variation in C_1 from C_0 to C_g and the linear variation in C_2 from C_0 to C_1/γ_c with variation in the density

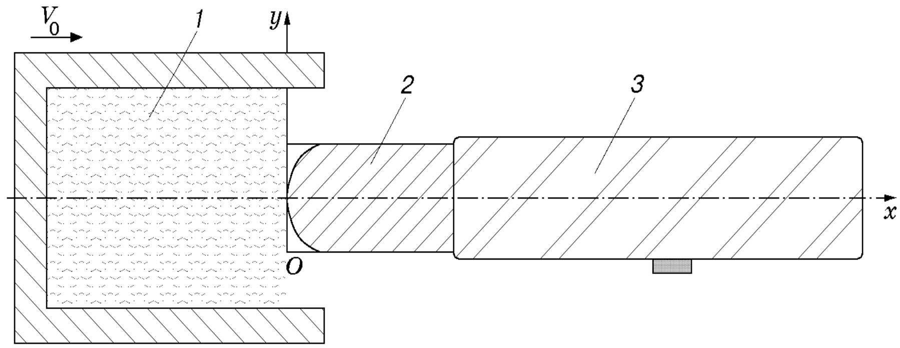

Fig. 2

ρ^* from ρ_0 to ρ_g. Here C_0 is the velocity of sound in the soil in the absence of disturbances (or at $\rho = \rho_0$). The stress deviator components are calculated on the basis of generalized Hooke's law

$$D_J s_{xx} + \lambda s_{xx} = 2G(2u_{,x} - v_{,y} - v/y)/3,$$

$$D_J s_{xy} + \lambda s_{xy} = G(u_{,y} + v_{,x}), \tag{8}$$

$$D_J s_{yy} + \lambda s_{yy} = 2G(2v_{,y} - u_{,x} - v/y)/3.$$

Here s_{ij} are the stress deviator components, $D_J s_{ij}$ is the Jaumann derivative with respect to time ($i, j = x, y, \theta$), and G is the shear modulus. The parameter λ can take values $\lambda = 0$ under purely elastic strain and $\lambda > 0$ if the plasticity condition is satisfied. The plasticity criterion for soil particles is specified by the relation [16]

$$J_2 = s_{ij}s^{ij}/2 = (kp + b)^2/6,$$

where J_2 is the second invariant of the stress-tensor deviator s_{ij} and k and b are specified constants. To describe the behavior of the casing, impactor head, and measuring rod, we used system (1), (8). A criterion of transition from the elastic stress–strain state to the plastic state for the container and the Hopkinson rod with the impactor is the Mises yield condition $J_2 = s_{ij}s^{ij}/2 = \sigma_{\rm y}^2/3$, where $\sigma_{\rm y}$ is the yield stress. The equation of state for spherical stress–tensor components is written as $p = K\varepsilon$, where K is the modulus of volume loading. The reduced equations and dependences correspond to the model of soil deformation developed by S. S Grigoryan.

Formulation of the Problem. The calculation scheme corresponds to the inverted experiment [13] and is shown in Fig. 2. The problem is formulated in the cylindrical coordinate system xOy. The Ox axis is the axis of symmetry passing through the rotation axes of the impactor head 2 and the measuring rod 3, and the Oy axis is perpendicular to it along the free surface of the soil 1. A difference method used is based on the modified scheme proposed by S. K. Godunov and is described in [21]. A distinctive feature of this method is that the Eulerian–Lagrangian approach was employed to describe the motion of media using arbitrary movable difference grids. To solve the problem formulated, we used the following conditions at the contact boundaries. On the free surface, $\sigma = 0$ and $\tau = 0$ (σ and τ are the normal and tangent stress components at the free boundary, respectively). On the surface of contact of the casing with the soil, we impose conditions of nonpenetration with ideal slip along the tangent: $v_{n1} = v_{n2}$ and $\tau_1 = \tau_2 = 0$ (v_n is the velocity component directed along the normal to the contact surface). On the surfaces of contact of the impactor with the rod and the soil, we impose conditions of nonpenetration along the normal $v_{n1} = v_{n2}$ and free slip in the tangential direction. Specifying the initial conditions, we assume that the impactor head and the measuring rod are in the unstressed state, and the rate of motion of the casing and the soil enclosed in it is V_0.

Calculation Results. In the inverted experiment, the target was sandy soil poured in a container of D16T aluminum alloy which had the following dimensions: length 70 mm, outside diameter 56.8 mm, inside diameter 54.5 mm, and bottom thickness 2 mm. The container was filled with the soil to a level of 65 mm. The impactor head was 20 mm long and 20 mm in diameter, and the measuring rod was 20.5 mm in diameter. The mechanical characteristics of the materials of the impactor head and the rod were the following: Young's modulus $E = 200$ and 145 GPa, yield stress $\sigma_{\rm y} = 400$ and 1200 MPa, and density $\rho = 7.8$ and 8.1 g/cm^3, respectively; Poisson's ratio for the material of the impactor head $\nu = 0.3$ and the velocity of sound in the rod $c = 4200$ m/sec. The

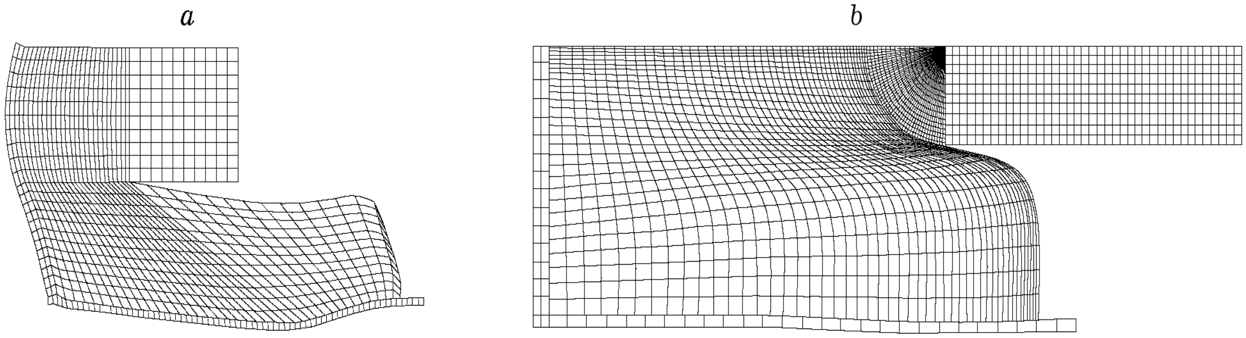

Fig. 3

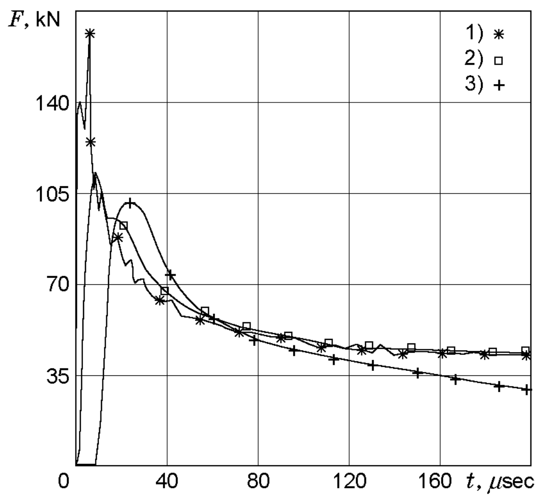

Fig. 4

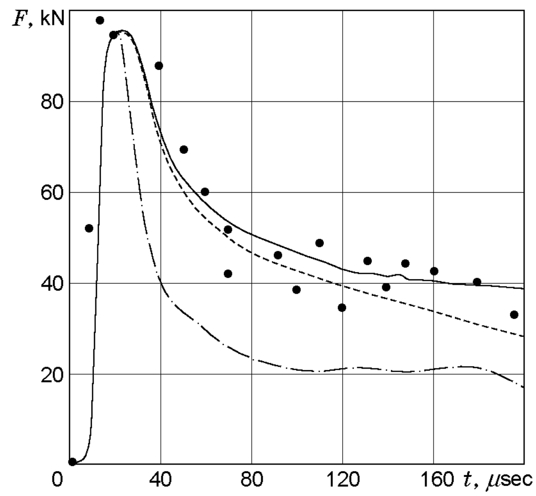

Fig. 5

experiments were performed with a dry compacted sand mixture of natural composition. In (3), $M = 2.1$ GPa and $n = 1.8$ [16]; in (4), $A = 500$ m/sec and $B = 2.41$ [17–19]. The values of the interpolating polynomial constants are $\alpha = \beta = 0.06$, $\rho_1 = 1.86$ g/cm^3, $\rho_4 = 2.15$ g/cm^3, $\gamma_c = 3$, and $\gamma_p = 4$. The initial density of the sand mixture was $\rho_0 = 1.76$ g/cm^3, $\rho_g = 2.62$ g/cm^3, the initial velocity of sound under unloading was $C_0 = 350$ m/sec, the shear modulus was $G = 100$ MPa, and the constants of the yield stress were $k = 1.25$ and $b = 0.5$ MPa [5]. The employed equations of state (3) and (5) correspond to those determined by the method of [15].

In this paper, we analyzed numerically the formation of an impulse during impact and its propagation in the rod. Numerical solution of the problem revealed that the impulse of force decays and "smears" with time, which is due to the large scheme viscosity of Godunov's method.

A half (in view of its axial symmetry) of the calculation region with a difference grid at the time $t = 200$ μsec is presented in Fig. 3. The figure shows the subregions occupied by the soil, casing, and the impactor head with the flat end. The calculation method used allowed us to describe the complex process of impactor penetration and compare the calculation results with the experimental data.

Figure 4 shows calculated curves of resistance forces versus time obtained by integration of the stress along the contact surface in the place of impact on the soil, in the place of contact with the measuring rod, and in the cross section located at a distance of three diameters from the rod end. Points 1 show values of the resistance force in the impact area, points 2 show these values in the place of contact of the impactor head with the rod, and points 3 show these values in the rod. Such transformation of the pulse is explained by plastic strains that arise at the specified impact velocity in the impactor head and decrease the maximum value of the stresses in the rod.

Figure 5 shows results of numerical simulation of the formulated problem in comparison with the experimental data. The points show values of the resistance force obtained experimentally, the solid curve shows a time curve of the resistance force in the rod at a distance of three diameters from the rod end for impact on infinite soil,

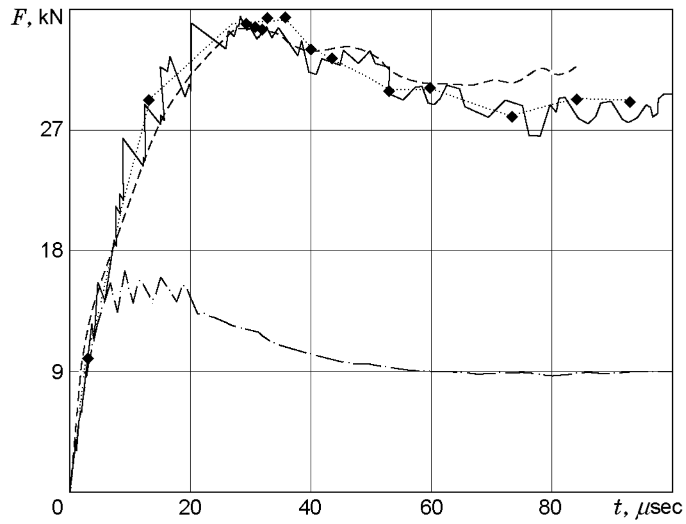

Fig. 6

and the dashed curve shows a time curve of the resistance force obtained in the numerical solution of the problem of impact on the soil in the casing (see [13]). The effect of volume compressibility of the soil on the force in the rod can be assessed by comparison of calculation results with allowance (solid and dashed curves in Fig. 5) and without allowance for volume unloading (dot-and-dashed curve). Numerical calculation results are in qualitative and quantitative agreement with the experimental data. It is seen that the casing and the volume plasticity of the soil have little effect on the maximum force value in the rod, and the difference in the amplitude is observed only with time.

We also studied nonstationary processes of interaction of a hemispherical impactor with the soil. The impact velocity was 223 m/sec. Figure 3b shows a half of the calculation region (except for the region occupied by the measuring rod) at the moment of completion of calculations $t = 100$ μsec.

Figure 6 shows calculated curves of the resistance force versus time obtained by integration of stresses along the contact surface in the place of impact on the soil (the solid curve refers to calculations and the points refer to the experiment).

Another important point is validation of the present method for determining the force of resistance to penetration by the strain pulse on the surface of the measuring rod. In Fig. 6, the dashed curve shows a time curve of the calculated resistance force in the cross section at a distance of three rod diameters from the rod end at an impact velocity of 223 m/sec. We also analyzed the effect of volume unloading on the resistance force. It was found that volume unloading does not affect the maximum value of the resistance force. The difference (to a lesser degree than indicated in [22]) is observed only after the maximum value is reached. The dot-and-dashed curve in Fig. 6 refers to the calculation results obtained under assumption that the behavior of the soil corresponds to the model of a nonlinearly compressed liquid. In the case of the impactor with a hemispherical head, the resistance force was more than half that for the cylindrical impactor with a flat edge.

An analysis of wave processes in the soil sample shows that by the completion of calculation, the compression wave reaches the container bottom only at velocities higher than 420 m/sec. The container walls have almost no effect on the resistance force, because a strong unloading wave is reflected from the free surface. The wave processes in the impactor head at a distance of 3–4 radii from the impact site can be considered one-dimensional, and the stress pulses propagate in the measuring rod without variation in temperature.

Conclusions. The impact of a hemispherical impactor on sand soil was simulated numerically using various models of soil deformation. Good agreement between calculation and experimental data is obtained for Grigoryan's model using the constants proposed for the equations of state in this paper. The calculated force of resistance to penetration and the force in the measuring rod are virtually equal. These results show the validity of recording the resistance force if the container sizes are three times as large as the measuring rod diameter. Thus, the inverted experimental technique considered is easy to implement and reasonably informative and can be used to determine integral stresses resulting from interaction of an impactor with soft soil.

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REFERENCES

1. N. A. Zlatin and G. I. Mishin (eds.), *Ballistic Facilities and Their Use in Experiments* [in Russian], Nauka, Moscow (1974).
2. F. F. Vitman and V. A. Stepanov, *Problems of Strength of Solids* [in Russian], Izd. Akad. Nauk SSSR, Moscow (1959).
3. A. Ya. Sagomonyan, *Dynamics of Target Penetration* [in Russian], Izd. Mosk. Univ., Moscow (1988).
4. W. Allen, E. Mayfield, and H. Morrison, "Dynamics of a projectile penetrating sand," *J. Appl. Phys*, **28**, 370–376 (1957).
5. G. E. Hauver, "Penetration with instrumented rods," *Int. J. Eng. Sci.*, **16**, No. 11, 871–877 (1978).
6. Yu. K. Bivin, Yu. M. Glukhov, and Yu. V. Permyakov, "Vertical penetration of solids into water," *Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza*, No. 6, 3–9 (1985).
7. D. K. Maynard, "Projectile impact and penetration dynamics. Correlation of analytical and experimental results," in: *Proc. Int. Conf. on Fract. Mech. and Tech.*, Part 1, Hong Kong (1977), pp. 1329–1341.
8. M. J. Forrestal, D. B. Longcope, and F. R. Norwood, "A model to estimate forces on conical penetrations into porous rock," *Trans. ASME, Ser. E, J. Appl. Mech.*, **48**, No. 3, 25–29 (1981).
9. C. O. Rogers and S. S. Pang, "Response of dry and liquid-filled porous rocks to static and dynamics loading by variously-shaped projectiles," *J. Rock Mech. Rock Eng.*, **19**, No. 4, 235–260 (1986).
10. Yu. N. Bukharev, V. P. Gandurin, and A. E. Korablev, "Experimental study of penetration of a rigid rod into clay soil and snow," in: *Applied Problems of Strength and Plasticity. Analysis and Optimization of Constructions* (collected scientific papers) [in Russian], No. 48, Izd. Nizh. Novgor. Univ., Nizhnii Novgorod (1991), pp. 99–106.
11. M. J. Forrestal, L. M. Lee, and B. D. Jenrette, "Laboratory-scale penetration experiments into geological targets to impact velocities of 2.1 km/sec," *Trans. ASME, Ser. E, J. Appl. Mech.*, **53**, No. 6, 317–320 (1986).
12. M J Forrestal, L. M. Lee, B. D. Jenrette, and R. E. Setchell, "Gas-gun experiments determine forces on penetrators into geological targets," *Trans. ASME, Ser. E, J. Appl. Mech.*, **51**, No. 6, 602–607 (1984).
13. V. V. Balandin and A. M. Bragov, "An experimental method for measuring resistance forces upon interaction of an impactor with soil," in: *Applied Problems of Strength and Plasticity. Methods of Solution* (collected scientific papers) [in Russian], No. 47, Izd. Nizh. Novg. Univ., Nizhnii Novgorod (1991), pp. 101–104.
14. N. A. Chichenev, A. B. Kudrin, and P. I. Polukhin, *Methods of Studying the Processes of Plastic Metal Working* [in Russian], Metallurgiya, Moscow (1977).
15. V. G. Bazhenov, A. M. Bragov, V. L. Kotov, et al., "Analysis of the applicability of a modified Kol'skii's method for dynamic tests of soils in a deformable casing," *Prikl. Mekh. Tekh. Fiz.*, **41**, No. 3, 155–162 (2000).
16. G. V. Rykov, "Experimental study of the stress field in an explosion in sandy soil," *Prikl. Mekh. Tekh. Fiz.*, No. 1, 85–89 (1964).
17. V. A. Lagunov and V. A. Stepanov, "Measurements of dynamic compressibility of sand under high pressures," *Prikl. Mekh. Tekh. Fiz.*, No. 1, 88–96 (1963).
18. M. D. Dianov, N. A. Zlatin, S. M. Mochalov, et al., "Impact compressibility of dry and water-saturated sand," *Pis'ma Zh. Tekh. Fiz.*, **2**, No. 12, 529–532 (1976).
19. A. M. Bragov and G. M. Grushevskii, "Effect of humidity and granulometric composition on the impact compressibility of sand," *Pis'ma Zh. Tekh. Fiz.*, **19**, No. 12, 70–72 (1993).
20. I. D. Faux and M. J. Pratt, *Computational Geometry for Design and Manufacture*, Ellis Horwood Ltd., New York (1979).
21. M. Kh. Abuzyarov, V. G. Bazhenov, V. L. Kotov, et al., "The method of decay of discontinuity in the dynamics of elastoplastic media," *Zh. Vychisl. Mat. Mat. Fiz*, **40**, No. 6, 940–953 (2000).
22. A. M. Bragov, V. L. Kotov, A. V. Kochetkov, and S. V. Krylov, "Inverted experiment and numerical analysis of axisymmetric processes of impact of solids on sandy soils," in: *Applied Problems of Strength and Plasticity. Numerical Simulation of Physico-Mechanical Processes* (collected scientific papers) [in Russian], **60**, Nauch. Izd. KMK, Moscow (1999), pp. 12–18.